eSpyMath: AP Precalculus Practice Questions & Answers (2024)

1. If α and β **are the roots of the quadratic equation of** $x^2 - 4x + 2 = 0$**, determine** $(\alpha+\beta)\times(\alpha\times\beta)$

Solution:

- By Vieta's formulas, $\alpha+\beta=4$ and $\alpha\times\beta=2$.
- $(\alpha+\beta)\times(\alpha\times\beta)=4\times2=8$

2. If α and β **are the roots of the quadratic equation of** $x^2 - 4x + 2 = 0$**, determine** $|\alpha-\beta|$

Solution:

- By Vieta's formulas, $\alpha+\beta=4$ and $\alpha\times\beta=2$.
- $|\alpha-\beta|=\sqrt{(\alpha+\beta)^2-4\alpha\beta} \; =\sqrt{4^2-4\times2}=2\sqrt{2}$

3. Find $f\circ g(4)$ **where** $f\left(\frac{1}{x}\right)=\sqrt{x}-2$ **and** $g(x)=2x$

Solution:

- First, find $g(4)$: $g(4)=2\times4=8$
- Then apply f to the result: $f(g(4))=f\left(\frac{1}{8}\right)=\sqrt{\frac{1}{8}}-2=\frac{1}{2\sqrt{2}}-2=\frac{\sqrt{2}}{4}-2$

4. For what value of a, the value of the term of $ax^2 - 2x + 3a$ **is always positive?**

Solution:

- For a quadratic equation $ax^2-2x+3a$ to be always positive, the discriminant must be negative, and .. a .. must be positive.
- The discriminant is $b^2-4ac=(-2)^2-4(a)(3a)=4-12a^2$. For it to be negative, $4-12a^2<0$, which simplifies to $a^2>\frac{1}{3}$. Since a must be positive, $a>\sqrt{\frac{1}{3}}$.

5. What is the numerical coefficient of the term containing u^4v^3 in the expansion of $(3u+v)^7$?

Solution:

- Use the binomial theorem to expand $(3u+v)^7$.
- The term containing u^4v^3 will have a binomial coefficient $\binom{7}{3}$ because we are choosing 3 vs from the 7 terms.
- Calculate the coefficient: $\binom{7}{3}\cdot(3u)^4\cdot v^3$.
- Compute the binomial coefficient: $\binom{7}{3}=\frac{7!}{3!(7-3)!}=35$.
- The numerical coefficient is $35\cdot 3^4=35\cdot 81=2835$.

6. What is the inverse of the matrix $\begin{bmatrix} 5 & -1 \\ 7 & 2 \end{bmatrix}$?

Solution:

- Find the determinant of the matrix $Det(A)=(5)(2)-(-1)(7)=10+7=17$.
- Compute the inverse matrix using the formula $\frac{1}{Det(A)}\times \text{Adj}(A)$, where Adj(A) is the adjugate of matrix A.
- Calculate Adj(A) by swapping the positions of a_{11} and a_{22}, and changing the signs of a_{12} and a_{21}: Adj(A) = $\begin{bmatrix} 2 & 1 \\ -7 & 5 \end{bmatrix}$.
- Multiply each element by $\frac{1}{17}$ to get the inverse: $\frac{1}{17}\begin{bmatrix} 2 & 1 \\ -7 & 5 \end{bmatrix}$.

7. If $g(x)=x^4+Cx^3+Dx^2+6$, and $g(2)=8$ and $g(-2)=-32$, what is the value of $3C+D$?

Solution:

- Substitute $x=2$ into the equation to get $g(2)=2^4+C(2)^3+D(2)^2+6=8$.

- This simplifies to $16+8C+4D+6=8$, which reduces to $8C+4D=-14$.
- Substitute $x=-2$ into the equation to get $g(-2)=(-2)^4+C(-2)^3+D(-2)^2+6=-32$.
- This simplifies to $16-8C+4D+6=-32$, which reduces to $-8C+4D=-54$.
- Solve the system of equations $\begin{cases} 8C+4D=-14 \\ -8C+4D=-54 \end{cases}$ to find C and D.
- Adding the two equations gives $8D=-68$, so $D=-8.5$.
- - Substitute D into one of the equations to find C : $8C=-14-4(-8.5)$, hence $8C=20$ and $C=2.5$.
- Compute $3C+D=3(2.5)-8.5=7.5-8.5=-1$.

8. If $h(x)=x^5-3x^3+2x-4$, find $h(-1)$.

Solution:

- Substitute $x=-1$ into the equation: $h(-1)=(-1)^5-3(-1)^3+2(-1)-4$.
- Simplify the equation: $h(-1)=-1+3-2-4$.
- Calculate the value: $h(-1)=-4$.

9. What is the domain of $h(x)=\dfrac{x^2-9}{x^3-27}$?

Solution:

- To find the domain, identify the values for which the denominator is zero since division by zero is undefined.
- Factor the denominator: $x^3-27=(x-3)(x^2+3x+9)$.
- Set the denominator equal to zero and solve: $x-3=0$, so $x=3$.
- The quadratic x^2+3x+9 does not have real roots (as its discriminant $b^2-4ac=3^2-4\cdot1\cdot9<0$).
- Therefore, the only value to exclude from the domain is $x=3$.
- The domain is all real numbers except $x=3$: $(-\infty,3)\cup(3,+\infty)$

10. What is the domain of $p(x)=\dfrac{\sqrt{2x-6}}{x^2-4x+4}$?

Solution:

- To find the domain, ensure the expression under the square root is non-negative and the denominator is not zero.
- - The expression under the square root, $2x-6$, must be ≥ 0. Solve for x : $2x-6\geq 0$ implies $x\geq 3$.
- Factor the denominator: $x^2-4x+4=(x-2)^2$.
- The denominator is zero when $x=2$. However, this is not in the range we found from the square root requirement.
- Therefore, the domain is $x\geq 3$ and $x\neq 2$. Since $x=2$ is not in the domain from the square root, it does not need to be excluded again.
- The domain is $x\geq 3$.

11. Which of the following represents the equation of $k^{-1}(x)$ for the inverse of the function $k(x)=2^{x+2}-4$?

Solution:

- To find the inverse, first replace $k(x)$ with y : $y=2^{x+2}-4$.
- Solve for x in terms of y : Add 4 to both sides to get $y+4=2^{x+2}$.
- Take the logarithm base 2 of both sides: $\log_2(y+4)=x+2$.
- Solve for x : $x=\log_2(y+4)-2$.
- Replace y with $k^{-1}(x)$ and x with y to get the inverse function: $k^{-1}(x)=\log_2(x+4)-2$.

12. Which of the following is equivalent to the expression $\log_5\left(\frac{p^4q^3}{r^2}\right)$?

(A) $\frac{4}{5}\log_5 p+\frac{3}{5}\log_5 q-\frac{2}{5}\log_5 r$

(B) $\frac{1}{5}(4\log_5 p+3\log_5 q)-\frac{2}{5}\log_5 r$

(C) $4\log_5 p+3\log_5 q-2\log_5 r$

(D) $\frac{4}{2}\log_5 p+\frac{3}{2}\log_5 q-\log_5 r$

Solution:

- Apply the logarithm properties: $\log_b(mn)=\log_b m+\log_b n$ and $\log_b\left(\frac{m}{n}\right)=\log_b m-\log_b n$
- Split the expression: $\log_5 p^4+\log_5 q^3-\log_5 r^2$.
- Apply the power rule: $4\log_5 p+3\log_5 q-2\log_5 r$.
- The correct answer is (C).

13. Identify the vertical asymptote(s) for the function $g(x)=\dfrac{x^3-1}{x^2-x-6}$.

Solution:

- Factor the denominator: $x^2-x-6=(x-3)(x+2)$.
- Set each factor equal to zero and solve for x : $x-3=0$ and $x+2=0$ gives $x=3$ and $x=-2$.
- The vertical asymptotes are where the denominator is zero: $x=3$ and $x=-2$.

14. Given $h(x)=\begin{cases}-2x^2+8, & x\le 1\\ \sqrt{x+4}, & x>1\end{cases}$, find $h(0.5)$.

Solution:

- Since $0.5\le 1$, we use the first piece of the piecewise function: $h(0.5)=-2(0.5)^2+8$.
- Compute the value: $h(0.5)=-2\cdot 0.25+8=-0.5+8=7.5$.

15. Find the slant asymptote of $m(x)=\dfrac{2x^2-5x}{x-2}$.

Solution:

- Perform polynomial long division or synthetic division to divide $2x^2-5x$ by $x-2$.
- The quotient will be the slant asymptote.
- The division yields $2x-1$ with a remainder that we discard for finding the asymptote.
- The slant asymptote is $y=2x-1$.

16. In polar coordinates, which of the following choices is not equivalent to $(3, -\frac{\pi}{4})$?

(A) $(3, \frac{7\pi}{4})$

(B) $(-3, \frac{3\pi}{4})$

(C) $(3, \frac{15\pi}{4})$

(D) $(-3, -\frac{5\pi}{4})$

Solution:

- The polar coordinate (r, θ) is equivalent to $(r, \theta + 2k\pi)$ for any integer k , and $(-r, \theta + \pi)$.
- For $(3, -\frac{\pi}{4})$, adding 2π to $-\frac{\pi}{4}$ repeatedly gives us equivalent angles in the positive direction like $\frac{7\pi}{4}$ and $\frac{15\pi}{4}$.
- Flipping the radius to -3 and adding π to the angle gives us $(-3, \frac{3\pi}{4})$.
- **Choice (D) is not equivalent** since $(-3, -\frac{5\pi}{4})$ would be equivalent to $(3, \frac{3\pi}{4})$ after adding π to the angle, which is different from the original angle $-\frac{\pi}{4}$.

17. Which of the following represents zeros of $S(\theta) = 3 - 3\cos 2\theta$?

(A) $\frac{\pi}{6}, \frac{5\pi}{6}$

(B) $0, \frac{\pi}{2}, \pi$

(C) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

(D) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

Solution:

- Set $S(\theta)=0$ to find the zeros: $3-3\cos 2\theta=0$.
- Simplify to find $\cos 2\theta=1$.
- The solutions for $\cos 2\theta=1$ are at $2\theta=2k\pi$ for any integer k , which gives $\theta=k\pi$.
- The zeros within one period $[0,2\pi)$ are 0 and π, which are reflected in option (B).

18. Which of the following choices represents the corresponding rectangular equation of the curve with the parametric equations $x(t)=4t$, $y(t)=t^2-2t+3$?

(A) $y=\frac{1}{16}x^2-\frac{1}{2}x+3$

(B) $y=\frac{1}{4}x^2-2x+3$

(C) $y=x^2-8x+12$

(D) $y=4x-x^2+3$

Solution:

- Eliminate the parameter t by expressing t in terms of x : since $x=4t$, we have $t=\frac{x}{4}$.
- Substitute t into $y(t)$: $y=(\frac{x}{4})^2-2(\frac{x}{4})+3$.
- Simplify the equation: $y=\frac{1}{16}x^2-\frac{1}{2}x+3$.
- The correct answer is (A).

19. Evaluate: $\sin\left(\arctan\left(\frac{4}{3}\right)\right)$**, given that** $0\le\theta\le\frac{\pi}{2}$**.**

Solution:

- Use the right triangle definition of tangent, where $\tan(\theta)=\frac{opposite}{adjacent}$.
- If $\tan(\theta)=\frac{4}{3}$, then a right triangle with angle θ can have sides of 4 (opposite), 3 (adjacent), and 5 (hypotenuse) by the Pythagorean theorem.

- Since $\sin(\theta) = \frac{opposite}{hypotenuse}$, then $\sin(\arctan(\frac{4}{3})) = \frac{4}{5}$.

20. Given that $\cos\alpha = \frac{3}{5}$ **and** $\sin\alpha < 0$**, find** $\sin\alpha$ **.**

(A) $\sqrt{\frac{16}{25}}$

(B) $-\sqrt{\frac{16}{25}}$

(C) $\frac{4}{5}$

(D) $-\frac{4}{5}$

Solution:

- Use the Pythagorean identity $\sin^2\alpha + \cos^2\alpha = 1$.
- Substitute $\cos\alpha = \frac{3}{5}$ into the identity and solve for $\sin\alpha$: $\sin^2\alpha = 1 - \left(\frac{3}{5}\right)^2$.
- Simplify to find $\sin^2\alpha$: $\sin^2\alpha = 1 - \frac{9}{25} = \frac{16}{25}$.
- Since $\sin\alpha < 0$, choose the negative root: $\sin\alpha = -\sqrt{\frac{16}{25}} = -\frac{4}{5}$.
- The correct answer is (D).

21. Which of the following is a possible equation for the sinusoidal graph shown with a maximum at $y = 4$**, a minimum at** $y = -2$**, and a period of 8 ?**

(A) $y = 3\sin\left(\frac{\pi}{4}x\right) + 1$

(B) $y = 3\cos\left(\frac{\pi}{4}x\right) - 1$

(C) $y = 6\sin\left(\frac{\pi}{4}x\right)+1$

(D) $y = 6\cos\left(\frac{\pi}{4}x\right)+1$

Solution:

- The amplitude of the graph is $\frac{4-(-2)}{2}=3$.
- The vertical shift is halfway between the maximum and minimum: $\frac{4+(-2)}{2}=1$.
- The period of the function is 8 , so the b value in $\sin(bx)$ or $\cos(bx)$ is $\frac{2\pi}{period}=\frac{2\pi}{8}=\frac{\pi}{4}$.
- Since the graph is a sine wave and the maximum is not at the y-axis, we use the sine function.
- The correct equation is $y=3\sin\left(\frac{\pi}{4}x\right)+1$ which reflects the amplitude and vertical shift.
- The correct answer is (A).

22. Which of the following is equivalent to $2sin(7x)cos(2x)+2cos(7x)sin(2x)$ **?**

(A) 2sin 5x

(B) 2sin 9x

(C) 2cos 5x

(D) 2cos 9x

Solution:

- We can use the sum-to-product formulas to simplify the expression:
- $2\sin(7x)\cos(2x)+2\cos(7x)\sin(2x)$
- Using the formula for the sine of a sum,
  - $2\sin A\cos B=\sin(A+B)+\sin(A-B)$
  - $2\cos A\sin B=\sin(A+B)-\sin(A-B)$

- So, $2\sin 7x\cos 2x + 2\cos 7x\sin 2x$
  $= (\sin(7x+2x)+\sin(7x-2x)) + (\sin(7x+2x)-\sin(7x-2x))$
  $= (\sin 9x + \sin 5x) + (\sin 9x - \sin 5x) = 2\sin 9x$
- So the equivalent expression is $2\sin 9x$, which is option (B).

23. Given cos x = -1/3 and tan x > 0, find cos 2x.

Solution:

- To find cos 2x, we can use the double-angle formula: $\cos 2x = 2\cos^2 x - 1$
- Given $\cos x = -\frac{1}{3}$, $\cos 2x = 2\left(-\frac{1}{3}\right)^2 - 1 = -\frac{7}{9}$
- Since tan x > 0 and cos x < 0, it means x is in the second quadrant, and cos 2x would indeed be negative.

24. Determine the period of the function $y = -3\cos\left(2\left(x+\frac{\pi}{4}\right)\right)$.

Solution:

- The period of a cosine function $\cos(Bx)$ is given by $\frac{2\pi}{|B|}$.
- For $y = -3\cos\left(2\left(x+\frac{\pi}{4}\right)\right)$, the period is: $\frac{2\pi}{|2|} = \frac{2\pi}{2} = \pi$

25. Solve the equation $\log_d(5-\log_d(m)) = n$ for m.

Solution:

- First, rewrite the equation in exponential form: $d^n = 5 - \log_d(m)$
- Now, solve for $\log_d(m)$: $\log_d(m) = 5 - d^n$
- Again, in exponential form: $m = d^{5-d^n}$
- This would be the solution for m in terms of d and n .

26. Determine whether the function $h(x) = x^6 - x^2\cos(x)$ is odd, even, or neither.

Solution:

- Test for even function: Check if $h(-x)=h(x)$
  - $h(-x)=(-x)^6-(-x)^2\cos(-x)=x^6-x^2\cos(x)=h(x)$
  - Therefore, the function is even.
- Test for odd function: Check if $h(-x)=-h(x)$
  - $h(-x)=(-x)^6-(-x)^2\cos(-x)=x^6-x^2\cos(x)\neq -h(x)$
  - Therefore, the function is not odd.
- Since the function is even, it is symmetric about the y-axis.

27. Find the linear equation for the function p that passes through the points $(2,-5)$ **and** $(-4,3)$**.**

Solution:

- Find the slope (m) using the two points:
  - $m=\dfrac{3-(-5)}{-4-2}=\dfrac{8}{-6}=-\dfrac{4}{3}$
- Use point-slope form with one of the points, say (2, -5):
  - $y-(-5)=-\dfrac{4}{3}(x-2)$
- Simplify the equation:
  - $y+5=-\dfrac{4}{3}x+\dfrac{8}{3}$
  - $y=-\dfrac{4}{3}x+\dfrac{8}{3}-\dfrac{15}{3}$
  - $y=-\dfrac{4}{3}x-\dfrac{7}{3}$

28. Find *Inverse fuction of* $f(x)$, which is $f^{-1}(x)$ where $f(x-1)=x^2+2x$

Solution:

- To find the inverse, we solve $f(x)=y=(x-1)^2+2(x-1)=x^2-1$ for x in terms of y and switching x and y. This requires rearranging the equation and solving the resulting quadratic equation for x.
- $f^{-1}(x)=\pm\sqrt{x-1}$

29. If $m(x) = x^3 + 1$ and $n(x) = \sqrt[3]{x-5}$, find the composition $(n^\circ m)(x)$.

Solution:

- Replace x in $n(x)$ with $m(x)$:
  - $(n^\circ m)(x) = n(m(x)) = \sqrt[3]{m(x)-5}$
- Substitute $m(x)$ into the expression:
  - $(n^\circ m)(x) = \sqrt[3]{(x^3+1)-5}$
- Simplify the composition:
  - $(n^\circ m)(x) = \sqrt[3]{x^3+1-5}$
  - $(n^\circ m)(x) = \sqrt[3]{x^3-4}$

30. Evaluate the limit $\lim_{x\to\infty} k(x)$, where $k(x) = (3x-5)^2 - (2x+7)^4$.

Solution:

- Identify the highest power of x in each term when $x \to \infty$.
- Compare the coefficients of the highest powers: $(3x)^2$ for the first term and $(2x)^4$ for the second term.
- The term $(2x)^4$ grows faster than $(3x)^2$ as $x \to \infty$.
- Since $(2x)^4$ dominates, the limit will be influenced by its sign, which is negative.
- Therefore, $\lim_{x\to\infty} k(x) = -\infty$.

31. Solve the inequality: $\dfrac{3(x+2)}{(x-2)(x+4)} \geq 0$.

Solution:

- Identify the critical points by setting the numerator and denominator equal to zero:
  - $3(x+2) = 0$ gives $x = -2$,
  - $x - 2 = 0$ gives $x = 2$,
  - $x + 4 = 0$ gives $x = -4$.
- Plot the critical points on a number line and test intervals to determine the sign of the expression.

- Combine the intervals where the expression is non-negative: $x \in (-4,-2] \cup (2,\infty)$

32. Determine the equation of the ellipse shown in the graph below, with its center at $(2,-3)$, horizontal major axis of length 10, and minor axis of length 6.

Solution:

- Find the lengths of the semi-major axis (a) and semi-minor axis (b):
  - $a = \frac{10}{2} = 5$,
  - $b = \frac{6}{2} = 3$.
- Use the standard form of the ellipse equation:
  - $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$.
- Substitute the center coordinates (h, k) and the values for a and b:
  - $\frac{(x-2)^2}{5^2} + \frac{(y+3)^2}{3^2} = 1$.
- Simplify the equation:
  - $\frac{(x-2)^2}{25} + \frac{(y+3)^2}{9} = 1$.

33. Evaluate the limit: $\lim_{x \to -\infty} 4^{-x} - 1$.

Solution:

- Recognize that as $x \to -\infty$, 4^{-x} approaches 0 because negative exponents represent reciprocals.
- Apply the limit to each term individually:
  - $\lim_{x \to -\infty} 4^{-x} = \infty$ and $\lim_{x \to -\infty} -1 = -1$.
- Combine the results:
  - $\lim_{x \to -\infty} 4^{-x} - 1 = 0 - 1$.
- Conclude the limit:
  - $\lim_{x \to -\infty} 4^{-x} - 1 = -1$.

34. What is the function whose graph is a reflection over the y-axis of the graph of $h(x) = 2 - 4^x$? (Reflection over the y-axis)

Solution:

- Step 1: Recognize that reflecting a graph over the y-axis means replacing x with $-x$.
- Step 2: Apply this transformation to the function $h(x)$ to get the reflection.
- Step 3: The new function will be $h(-x) = 2 - 4^{-x}$.
- Step 4: Simplify the function to $g(x) = 2 - \frac{1}{4^x} = 2 - 4^{-x}$.

35. Which of the following functions does not have an inverse function on the specified domain?

(A) $y = \cos(x)$, where $0 \le x \le \pi$

(B) $y = x^2 - 3$

(C) $y = \frac{1}{x} - 3$

(D) $y = 3^x$

Solution:

- Step 1: Check if each function is one-to-one on the specified domain.
- Step 2: $y = \cos(x)$ is one-to-one on $[0, \pi]$.
- Step 3: $y = x^2 - 3$ is not one-to-one because it is a quadratic function and fails the horizontal line test.
- Step 4: $y = \frac{1}{x} - 3$ is one-to-one on its domain.
- Step 5: $y = 3^x$ is one-to-one because exponential functions are always one-to-one.
- Step 6: The function without an inverse is $y = x^2 - 3$.

36. Give an algebraic expression for $\sin(\cos^{-1}(x))$. (Algebraic Expression)

Solution:

- Step 1: Let $\theta = \cos^{-1}(x)$, so $\cos(\theta) = x$ and $\sin^2(\theta) + \cos^2(\theta) = 1$.

- Step 2: Solve for $\sin^2(\theta)$: $\sin^2(\theta) = 1 - \cos^2(\theta) = 1 - x^2$.
- Step 3: Since $\sin(\theta)$ will be positive for θ in the range of $\cos^{-1}$, take the positive square root: $\sin(\theta) = \sqrt{1 - x^2}$.

37. A circle is graphed using the parametric equations shown below: $x = 7\sin(t) - 2$ and $y = 7\cos(t) + 4$ **Where is the center of the circle located? (Parametric Equations)**

Solution:

- Step 1: Recognize that the center of the circle in parametric form $(x, y) = (r\sin(t) + h, r\cos(t) + k)$ is (h, k).
- Step 2: Identify h and k from the given equations.
- Step 3: The center of the circle is at $(-2, 4)$.

38. The table shows the predicted growth of a particular bacteria population after various numbers of hours. Write an explicit formula for the sequence of the number of bacteria. (Arithmetic Sequence)

Hours (n)	**1**	**2**	**3**	**4**	**5**
Bacteria (b_n)	23	46	69	92	115

Solution:

- Step 1: Notice the common difference between consecutive numbers of bacteria is 23.
- Step 2: Recognize that this represents an arithmetic sequence with a common difference $d = 23$.
- Step 3: Find the first term of the sequence $a_1 = 23$.
- Step 4: Use the formula for the nth term of an arithmetic sequence $b_n = a_1 + (n-1)d$.
- Step 5: Substitute a_1 and d into the formula to get $b_n = 23 + (n-1) \times 23$.
- Step 6: Simplify to get the explicit formula $b_n = 23n$.

39. What are the points where the graph of the polynomial $g(x) = 3(x+6)(x-6)^2$ **crosses the x-axis? (Polynomial Zeros)**

Solution:

- Step 1: Set the polynomial equal to zero to find the roots: $0 = 3(x+6)(x-6)^2$.
- Step 2: Solve for x by finding the values that make each factor zero.
- Step 3: The first factor is zero when $x = -6$.
- Step 4: The second factor is zero when $x = 6$, but it's squared, meaning the graph only touches the x-axis at this point.
- Step 5: The points where the graph crosses the x-axis are $x = -6$ and $x = 6$ (the latter is a double root, but for crossing, we count it once).

40. Evaluate: $\lim_{x \to \infty} \frac{2x^2 - 5x - 7}{4x^2 + 5x - 9}$ (Limits)

Solution:

- Step 1: Identify the highest power of x in the numerator and denominator (which is x^2).
- Step 2: Divide every term by x^2 to get $\lim_{x \to \infty} \frac{2 - \frac{5}{x} - \frac{7}{x^2}}{4 + \frac{5}{x} - \frac{9}{x^2}}$.
- Step 3: As x approaches infinity, the terms with x in the denominator approach zero.
- Step 4: After simplifying, the limit becomes $\frac{2}{4}$.
- Step 5: Reduce the fraction to get the final answer $\frac{1}{2}$.

41. Given the function $g(x) = (x^2 - 4) - (x-2)(x+1)$, at what value of x is the absolute maximum of $g(x)$ over the interval $[-1.5, 3]$? (Maximum Value of a Function)

Solution:

- Step 1: Expand $g(x)$ to get $g(x) = x^2 - 4 - (x^2 - x - 2)$.
- Step 2: Simplify the function: $g(x) = x - 2$.
- Step 3: Recognize that this is a linear function, which means the maximum value will occur at one of the endpoints of the interval.
- Step 4: Evaluate $g(-1.5)$ and $g(3)$.
- Step 5: $g(-1.5) = -3.5$ and $g(3) = 1$.
- Step 6: The maximum value is 1 at $x = 3$.

42. Approximate $\log_5 36$, given that $\log_2 5 \approx 2.33$ and $\log_2 3 \approx 1.58$.

Solution:

- Step 1: Express $\log_5 36$ in terms of $\log_2$ using the change of base formula: $\log_5 36 = \frac{\log_2 36}{\log_2 5}$.
- Step 2: Recognize that $\log_2 36 = \log_2(2^2 \cdot 3^2) = 2 + 2\log_2 3$.
- Step 3: Substitute the approximate values: $\log_5 36 = \frac{2 + 2\log_2 3}{\log_2 5} = \frac{2 + 2(1.58)}{2.33} \approx 2.21$.

43. (Exponential Equation): Solve for y : $5^{2y} = 25^{y+1}$.

Solution:

- Step 1: Recognize that 25 is 5^2.
- Step 2: Rewrite the equation: $5^{2y} = (5^2)^{y+1}$.
- Step 3: Simplify the right side: $5^{2y} = 5^{2y+2}$.
- Step 4: Since the bases are the same, set the exponents equal to each other: $2y = 2y + 2$.
- Step 5: Solve for y : this results in a contradiction, indicating there is **no solution**.

44. Which of the following choices is equivalent to the complex number $1 - 4i$? (Complex Number Representation)

Solution:

- Step 1: Calculate the magnitude $r = \sqrt{1^2 + (-4)^2} = \sqrt{17}$.
- Step 2: Determine the angle $\theta = \arctan\left(\frac{b}{a}\right) = \arctan\left(\frac{-4}{1}\right) = 1.3258$ radians.
- Step 3: Express the complex number in polar form: $r(\cos(\theta) + i\sin(\theta))$.
- Step 4: Choose the option that matches $\sqrt{17}(\cos(-1.3258) + i\sin(-1.3258))$.

45. Find the angle between two vectors $\vec{r}$ and $\vec{s}$ with magnitudes 3 and 4, respectively, that has a dot product equal to 6. (Vector Dot Product)

Solution:

- Step 1: Use the dot product formula $\vec{r} \cdot \vec{s} = |\vec{r}||\vec{s}|\cos(\theta)$.
- Step 2: Substitute the given magnitudes and dot product into the formula: $6 = 3 \times 4 \times \cos(\theta)$.
- Step 3: Simplify to find $\cos(\theta) = \frac{6}{12} = \frac{1}{2}$.
- Step 4: Use the inverse cosine function to find $\theta = \cos^{-1}\left(\frac{1}{2}\right)$.
- Step 5: Determine that $\theta = \frac{\pi}{3}$ or 60 degrees.

46. How long will it take for 500 to triple in value in an investment when interest is <u>compounded continuously</u> at the rate of 4.2% per annum? Round your answer to the nearest year. (Compound Interest)

Solution:

- Step 1: Use the continuous compound interest formula $A = Pe^{rt}$, where A is the amount, P is the principal, r is the rate, and t is the time.
- Step 2: Plug in the values $A = 3P$, $P = 500$, and $r = 0.042$.
- Step 3: Solve for t in the equation $1500 = 500e^{0.042t}$.
- Step 4: Simplify to get $3 = e^{0.042t}$.
- Step 5: Take the natural logarithm of both sides to solve for t : $t = \frac{\ln(3)}{0.042}$.
- Step 6: Calculate t: t=26.15 and round to the nearest year t=26.

47. (Projectile Motion): A rock is thrown vertically upward from a cliff that is 100 feet above ground level with an initial velocity of 60 feet per second. The height h , in feet above ground level, of the rock t seconds after the throw is given by the function $h(t) = -16t^2 + 60t + 100$. At approximately what value of t will the rock be at the height of 80 feet and moving upward?

Solution:

- Step 1: Set the height function equal to 80 feet: $80 = -16t^2 + 60t + 100$.
- Step 2: Rearrange to form a quadratic equation: $-16t^2 + 60t + 20 = 0$.
- Step 3: Solve for t using the quadratic formula or factoring, if possible.
- Step 4: Identify the time t when the rock is moving upward by selecting the smaller positive root.

48. (Exponential Decay): A ball is dropped from a height of 25 feet. After each bounce, the ball reaches 80% of its previous height. How high will the ball rebound after the second bounce?

Solution:

- Step 1: Determine the height reached after the first bounce: 25 \times 0.8 .
- Step 2: Calculate the height after the first bounce: $25 \times 0.8 = 20$ feet.
- Step 3: Determine the height reached after the second bounce: 20×0.8.
- Step 4: Calculate the height after the second bounce: $20 \times 0.8 = 16$ feet.

49. (Exponential Decay): Suppose you release a balloon from a height of 20 feet. After it ascends, it stabilizes at 120% of its previous height each minute. How high, to the nearest tenth, will the balloon be after 2 minutes?

Solution:

- Step 1: Determine the height reached after the first minute: 20×1.2.
- Step 2: Calculate the height after the first minute: $20 \times 1.2 = 24$ feet.
- Step 3: Determine the height reached after the second minute: 24×1.2.
- Step 4: Calculate the height after the second minute: $24 \times 1.2 = 28.8$ feet.
- Step 5: Round to the nearest tenth to get 28.8 feet.

50. What is the third term in the expansion of $(2a+3b)^5$?

Solution:

- Step 1: The third term in the binomial expansion can be found using the formula $C(n,k) \cdot a^{n-k} \cdot b^k$ where n is the power of the binomial, a and b are the terms of the binomial, and k is the term number minus one.
- Step 2: For the third term, $k = 2$.
- Step 3: $C(5,2) \cdot (2a)^{5-2} \cdot (3b)^2 = 10 \cdot 8a^3 \cdot 9b^2 = 720a^3b^2$.

51. Simplify: $\ln\left(\sqrt[3]{e^3 x}\right)$.

Solution:

- Step 1: Recognize that $\ln(e^k) = k$ for any k .
- Step 2: Apply the property that $\ln(a^b) = b \cdot \ln(a)$.
- Step 3: $\ln\left((e^{3x})^{1/3}\right) = \frac{1}{3}\ln(e^{3x})$.
- Step 4: Simplify to get x .

52. If $\sec x \neq 1$, which of the following is equivalent to $\frac{\tan^2 x}{1+\sec x}$?

Solution:

- Step 1: Use the identity $\sec x = \frac{1}{\cos x}$ and $\tan^2 x = \sec^2 x - 1$ to rewrite the fraction.
- Step 2: $\frac{\sec^2 x - 1}{1+\sec x}$.
- Step 3: Recognize that $\sec^2 x - 1$ is the numerator of a difference of squares.
- Step 4: Factor and simplify: $\frac{(\sec x - 1)(\sec x + 1)}{1+\sec x}$.
- Step 5: Cancel out the common terms to get $\sec x - 1$.

53. 4. If $\cos\alpha = a$, then what is $\sin\alpha \cdot \cos\alpha \cdot \cot\alpha$?

Solution:

- Step 1: Use the identity $\sin^2\alpha + \cos^2\alpha = 1$ to find $\sin\alpha$.
- Step 2: Since $\cos\alpha = a$, $\sin\alpha = \sqrt{1-a^2}$.
- Step 3: $\cot\alpha = \frac{\cos\alpha}{\sin\alpha}$.
- Step 4: Plug in the values: $\sqrt{1-a^2} \cdot a \cdot \frac{a}{\sqrt{1-a^2}}$.
- Step 5: Simplify to get a^2.

54. If $g(\theta) = m\sin\theta + n$, what is the maximum value of $g(\theta)$?

Solution:

- The maximum value of $g(\theta)$ occurs when $\sin\theta$ is at its maximum, which is 1. Therefore, the maximum value of $g(\theta)$ is $m(1)+n=m+n$.

55. Which of the following equations could represent the graph shown below, assuming the graph depicts a simple transformation of a basic trigonometric function with a period of π and an amplitude of 1?

Solution:

- The graph's period is π, which means the basic trigonometric function must be altered to fit this period.
- Since the amplitude is 1, we do not need to modify the coefficient of the sine or cosine function for amplitude.
- Taking into account the reflection over the x-axis as seen in the original graph, the general form of the equation should be $y=-\sin(2x)$ or $y=-\cos(2x)$, depending on the starting point of the graph (at $y=0$ or a peak/trough).

56. Which of the following expressions is equivalent to $\sec\left(\frac{3\pi}{4}\right)$?

(A) $y=-\frac{1}{2}\cos(2x)$

(B) $\tan\frac{3\pi}{4}$

(C) $\csc\left(-\frac{3\pi}{4}\right)$

(D) $\cos\frac{3\pi}{4}$

Solution:

- To find the equivalent expression for $\sec\left(\frac{3\pi}{4}\right)$, we need to consider the function values and signs in the quadrants of the unit circle.

- For $\frac{3\pi}{4}$, $\sec\theta$ is the reciprocal of $\cos\theta$, and since $\cos\left(\frac{3\pi}{4}\right)$ is negative, the correct choice should also yield a negative value in the second quadrant where $\frac{3\pi}{4}$ lies.

57. In which quadrant is the terminal side of angle ϕ located if the graphs of $y = \tan\phi$ and $y = \sec\phi$ are both increasing when angle ϕ is increasing?

Solution:

- The tangent and secant functions are both positive in the third quadrant, but since the specifies they're increasing, the correct quadrant would be the one where both derivatives are positive.
- For tangent, the derivative is $\sec^2\phi$, which is always positive, and for secant, the derivative is $\sec\phi\tan\phi$, which is positive where secant is positive.

58. Evaluate: $\sec\left(\tan^{-1}\left(\frac{5}{12}\right)\right)$

Solution:

- To evaluate $\sec\left(\tan^{-1}\left(\frac{5}{12}\right)\right)$, you would create a right triangle with the opposite side of 5 units, an adjacent side of 12 units, and use the Pythagorean theorem to find the hypotenuse. Then $\sec$ of that angle is the hypotenuse over the adjacent side.

59. For the expression $k - \frac{1}{\cos^2\phi} = \cos^2\phi$ to be an identity, what does k equal?

(A) 1

(B) 0

(C) $\sin^2\phi$

(D) $\tan^2\phi$

Solution:

- For $k-\frac{1}{\cos^2\phi}=\cos^2\phi$ to be an identity, we can rewrite $\frac{1}{\cos^2\phi}$ as $\sec^2\phi$.
- By rearranging the terms and using the Pythagorean identity $\sec^2\phi=1+\tan^2\phi$, we find the value of k .

60. What is the expression $\frac{\cos 2\phi}{2\sin\phi}$ equivalent to?

Solution:

- The expression $\frac{\cos 2\phi}{2\sin\phi}$ can be simplified by using the double-angle formula for cosine, $\cos 2\phi=1-2\sin^2\phi$ or $\cos 2\phi=2\cos^2\phi-1$, and then dividing by $2\sin\phi$ to simplify to a single trigonometric function.

61. In the interval $0\le x<2\pi$, what are the solutions of the equation $\cos^2 x=\cos x$?

Solution:

- Step 1: Factor the equation: $\cos x(\cos x-1)=0$.
- Step 2: Solve for x : The solutions occur when $\cos x=0$ or when $\cos x=1$.
- Step 3: Find the x-values within the given interval where these conditions are true.

62. What is the expression $\frac{\cos(x-\frac{\pi}{2})}{\sin x}$ equivalent to?

Solution:

- Step 1: Use the identity $\cos(x-\frac{\pi}{2})=\sin x$ to rewrite the numerator.
- Step 2: Simplify the expression with the new numerator.

63. If $\cos C=\frac{4}{5}$ and $\cos D=\frac{5}{13}$, and if C and D are acute angles, what is the value of $\sin(C+D)$?

Solution:

- Step 1: Use the Pythagorean identity to find $\sin C$ and $\sin D$.
- Step 2: Use the sine addition formula: $\sin(C+D) = \sin C \cos D + \cos C \sin D$.
- Step 3: Plug in the values for $\sin C$, $\sin D$, $\cos C$, and $\cos D$ to find $\sin(C+D)$.

64. Which of the following choices represents the graph of $r = 4\sin\theta$ in polar coordinates?

(A) A circle centered at the pole with a radius of 4.

(B) A cardioid that starts at the pole and extends to the right.

(C) A limaçon with an inner loop.

(D) A circle centered on the horizontal axis, 2 units to the right of the pole.

Solution:

- The equation $r = 4\sin\theta$ describes a circle in polar coordinates with a diameter of 4 units, and because it's $\sin\theta$, it will be centered on the horizontal axis (since the sine function is associated with the y-coordinate in Cartesian coordinates).
- The circle will be above the horizontal axis, starting at the pole (origin), because $\sin\theta$ is positive in the first and second quadrants. The correct answer is (D).

65. Which of the following points does not change the location of the point $(3, \frac{3\pi}{4})$ in polar coordinates?

(A) $\left(3, \frac{11\pi}{4}\right)$

(B) $\left(-3, \frac{7\pi}{4}\right)$

(C) $\left(-3, \frac{3\pi}{4}\right)$

(D) $\left(3, \frac{-5\pi}{4}\right)$

Solution:

- In polar coordinates, the point (r,θ) is determined by the radius r and the angle θ. However, due to the periodic nature of angles, an angle can be increased or decreased by full rotations (multiples of 2π) without changing the location of the point: The given point is $(3,\frac{3\pi}{4})$.
- We need to determine which option, when the angle is adjusted by a multiple of 2π, will result in the point being in the same location.
- Option (A) $\left(3,\frac{11\pi}{4}\right)$, as it is the only option that has a positive radius and the angle differs from the original angle by a multiple of 2π.

66. Given the polar coordinates $(5,-\frac{\pi}{6})$, find the rectangular coordinates of this point.

Solution:

- The polar coordinates (r,θ) can be converted to rectangular coordinates (x,y) using the following formulas: $x = r\cos(\theta)$ and $y = r\sin(\theta)$
- Given polar coordinates $(5,-\frac{\pi}{6})$, we apply these formulas: $x = 5\cos(-\frac{\pi}{6})$ and $y = 5\sin(-\frac{\pi}{6})$
- Since $\cos(-\frac{\pi}{6}) = \cos(\frac{\pi}{6}) = \sqrt{3}/2$ and $\sin(-\frac{\pi}{6}) = -\sin(\frac{\pi}{6}) = -1/2$, we get: $x = 5\times\frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{2}$ and $y = 5\times\left(-\frac{1}{2}\right) = -\frac{5}{2}$
- So the rectangular coordinates are $(\frac{5\sqrt{3}}{2},-\frac{5}{2})$.

67. Given the rectangular coordinates (2, -2), find the polar coordinates of this point.

Solution:

- To convert the rectangular coordinates (x,y) to polar coordinates (r,θ), we use the formulas: $r = \sqrt{x^2 + y^2}$ and $\theta = \arctan\left(\frac{y}{x}\right)$

- Given rectangular coordinates $(2,-2)$, let's calculate: $r=\sqrt{2^2+(-2)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$ and $\theta=\arctan\left(\frac{-2}{2}\right)=\arctan(-1)$
- Since both x and y are negative, the point lies in the third quadrant. Therefore, θ should be $\pi+\arctan(-1)$, which is $\pi-\frac{\pi}{4}=\frac{3\pi}{4}$.
- The polar coordinates are $(2\sqrt{2},\frac{3\pi}{4})$.

68. If $h(x)=x^2-4x+3$ and $p(x)=3-x$, then what does $p(h(x))$ equal?

(A) $3x^2-12x+6$

(B) x^2-4x

(C) $-x^2+4x$

(D) $9-x^2+4x-3$

Solution:

- Given $h(x)=x^2-4x+3$ and $p(x)=3-x$, we need to compute $p(h(x))$. This means we replace every x in $p(x)$ with $h(x): p(h(x))=3-(x^2-4x+3)$

  $p(h(x))=3-x^2+4x-3$

  $p(h(x))=-x^2+4x$
- So the answer is $(C)-x^2+4x$.

69. Let g be a function defined for all real numbers. Which of the following conditions is not sufficient to guarantee that g has an inverse function?

(A) g is one-to-one.

(B) g has a continuous, non-repeating range.

(C) g has no critical points.

(D) g passes the Horizontal Line Test in its domain.

Solutions:

- For a function to have an inverse, it must be one-to-one (injective), which means it passes the Horizontal Line Test. This eliminates (A) and (D).
- (B) A continuous, non-repeating range is a necessary but not a sufficient condition for a function to have an inverse. A function can have a continuous, non-repeating range but not be one-to-one.
- (C) A function not having critical points does not ensure it is one-to-one. A function could have no critical points and still be a horizontal line, which would not be one-to-one.
- The correct answer is (C) g has no critical points. It is not a sufficient condition to guarantee an inverse because, without being one-to-one, a function cannot have an inverse, regardless of whether it has critical points or not.

70. Which of the following functions is not even?

(A) $q(x) = \cos(x)$

(B) $r(x) = \cos(3x)$

(C) $s(x) = x^6$

(D) $t(x) = \dfrac{x^3}{x^2 + 1}$

Solutions:

An even function is defined by the property $f(x) = f(-x)$. This means that the function is symmetric about the y-axis.

Let's check each function to see if it is even:

- (A) $q(x) = \cos(x)$: Cosine is an even function, meaning that $\cos(x) = \cos(-x)$. So, $q(x)$ is even.
- (B) $r(x) = \cos(3x)$: Again, cosine is an even function, and since the argument of the cosine function is simply scaled, this does not affect its evenness. Thus, $r(x)$ is even.
- (C) $s(x) = x^6$: Any function with an even power is an even function, since $(-x)^{2n} = x^{2n}$ where n is an integer. Therefore, $s(x)$ is even.

- (D) $t(x)=\frac{x^3}{x^2+1}$: Let's evaluate $t(-x)$: $t(-x)=\frac{(-x)^3}{(-x)^2+1}=\frac{-x^3}{x^2+1}$: As we can see, $t(x)\neq t(-x)$, which means that $t(x)$ is not an even function.

Therefore, the function that is not even is:(D) $t(x)=\frac{x^3}{x^2+1}$

71. At what value(s) of x do the graphs of $y=3x+1$ and $y^2=9-x^2$ intersect?

Solution:

- The two equations are: $y=3x+1$ and $y^2=9-x^2$
- To find their intersection points, set the expressions for y equal to each other:

$(3x+1)^2=9-x^2$

$9x^2+6x+1=9-x^2$

$10x^2+6x-8=0$

$5x^2+3x-4=0$

- To solve this quadratic equation, we can use the quadratic formula: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
- Where $a=5$, $b=3$, and $c=-4$: $x=\frac{-3\pm\sqrt{3^2-4\cdot5\cdot(-4)}}{2\cdot5}$

$x=\frac{-3\pm\sqrt{9+80}}{10}$; $x=\frac{-3\pm\sqrt{89}}{10}$

- So, the graphs intersect at the x-values: $x=\frac{-3+\sqrt{89}}{10}$ and $x=\frac{-3-\sqrt{89}}{10}$

72. If $g^{-1}(x)$ is the inverse of $g(x)=10^x$, then what does $g^{-1}(x)$ equal?

Solutions:

- Given the function g(x) = 10^x , to find its inverse $g^{-1}(x)$, we exchange x and y and solve for y : $x=10^y$
- Taking the logarithm (base 10) of both sides: $\log_{10}(x)=y$

- Thus, the inverse function $g^{-1}(x)$ is: $g^{-1}(x)=\log_{10}(x)$

73. Find $g(x+k)$ when $g(x)=3x^2+5x+2$.

Solutions:

- We have the function $g(x)=3x^2+5x+2$. To find $g(x+k)$, we substitute $x+k$ for x in the equation: $g(x+k)=3(x+k)^2+5(x+k)+2$
- Expanding the square: $g(x+k)=3(x^2+2xk+k^2)+5x+5k+2$
- Now combine like terms: $g(x+k)=3x^2+(6k+5)x+(3k^2+5k+2)$

74. Given the Markov matrix below, which of the following statements is true about the variable b ?

$$\begin{bmatrix} b & 1-b & 0 \\ 0 & b & 1-b \\ 1-b & 0 & b \end{bmatrix}$$

(A) b is any real number

(B) b is any positive real number

(C) $0\le b\le 1$

(D) $b=1$

Solutions:

A Markov matrix, also known as a stochastic matrix, is a square matrix used to describe the transitions of a Markov chain. Each element of the matrix represents the probability of transitioning from one state to another, with a couple of important properties:

1. Each element in the matrix must be between 0 and 1, inclusive, because they represent probabilities.

2. Each column of the matrix must sum to 1 because the probabilities of transitioning from any given state to all possible states must be certain (100%).

Given the matrix: $\begin{bmatrix} b & 1-b & 0 \\ 0 & b & 1-b \\ 1-b & 0 & b \end{bmatrix}$

We must check if it satisfies these properties for each option:

(A) b is any real number: This cannot be correct because if b were any real number, then some elements could be negative or greater than 1, which is not allowed for probabilities.

(B) b is any positive real number: This is also incorrect because even if b is positive, if it's greater than 1, then some elements could be negative, since the matrix includes $1-b$.

(C) $0 \le b \le 1$: This is a reasonable statement because it ensures that all elements are between 0 and 1, inclusive. However, we also need to check if the columns sum to 1.

For the first column: $b+(1-b)=1$

For the second column: $(1-b)+b=1$

For the third column: $0+(1-b)+b=1$

With $0 \le b \le 1$, each column sums to 1, satisfying the conditions for a Markov matrix.

(D) b = 1: If $b=1$, the matrix would become: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

While this is a valid stochastic matrix (it's actually the identity matrix which represents that the state stays the same with probability 1), it's too specific and doesn't represent the general condition needed for a matrix to be a Markov matrix.

The correct answer that fulfills the conditions of a Markov matrix is: (C) $0 \le b \le 1$

75. For $G(x)=g(x)+2$ where $g(x)=\begin{cases} 9x-2, & \text{if } x<3 \\ 5-x, & \text{if } x \ge 3 \end{cases}$, evaluate G(7) .

(A) 62

(B) -2

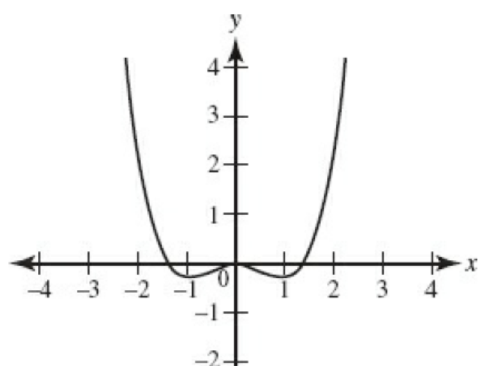
(C) 0

(D) 5

Solution:

- To evaluate $G(7)$, we first need to determine the correct piece of the piecewise function $g(x)$ to use for $x = 7$, and then add 2 to the result.
- Since $7 \geq 3$, we use the second piece: $g(7) = 5 - 7 = -2$
- Now, apply the definition of $G(x): G(7) = g(7) + 2 = -2 + 2 = 0$
- So, $G(7)$ evaluates to 0 , which means the correct answer is: (C) 0

76. Use the graph of $f(x)$ below to determine the intervals where $f(x)$ is increasing and where $f(x)$ is decreasing. Find correct one.



(A) Increasing: $(-2,1)$; decreasing: $(-\infty,-2) \cup (1,\infty)$

(B) Increasing: $(-2,0) \cup (1,\infty)$; decreasing: $(-\infty,-2) \cup (0,1)$

(C) Increasing: $(-1,\infty)$; decreasing: $(-\infty,-1)$

(D) Increasing: $(-\infty,-2) \cup (1,\infty)$; decreasing: $(-2,1)$

Solution: (c)

77. Using the tables below, evaluate $(h \circ f)(7)$. The tables provide values for functions f and h for various inputs.

x	2	7	9	11
f(x)	5	3	8	6

x	3	5	8	6
h(x)	12	14	7	15

(A) 12

(B) 14

(C) 7

(D) 15

Solution:

- To solve this, you would find the value of $f(7)$ from the first table and then use that result to find $h(\text{that result})$ from the second table. Answer is (A)

78. Find the matrix product CD if it is defined, given that $C=\begin{bmatrix} 2 & -1 \\ 4 & 6 \end{bmatrix}$ **and** $D=\begin{bmatrix} -1 & 2 \\ 3 & 5 \end{bmatrix}$**.**

Solution:

- To find the matrix product CD , you multiply matrix C by matrix D .
- $CD=\begin{bmatrix} 2 & -1 \\ 4 & 6 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 3 & 5 \end{bmatrix}$
- $CD=\begin{bmatrix} (2\cdot -1)+(-1\cdot 3) & (2\cdot 2)+(-1\cdot 5) \\ (4\cdot -1)+(6\cdot 3) & (4\cdot 2)+(6\cdot 5) \end{bmatrix}=\begin{bmatrix} -2-3 & 4-5 \\ -4+18 & 8+30 \end{bmatrix}=\begin{bmatrix} -5 & -1 \\ 14 & 38 \end{bmatrix}$

79. Calculate the area of the triangle with the vertices $(1,1)$**,** $(4,3)$**, and** $(6,7)$**.**

Solution:

- The area A of a triangle given vertices (x_1,y_1), (x_2,y_2), and (x_3,y_3) can be found using the following formula: $A=\frac{1}{2}|\,x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\,|$

- Using the given points: $A = \frac{1}{2} | 1(3-7) + 4(7-1) + 6(1-3) |$

- $A = \frac{1}{2} | -4 + 24 - 12 | = 4$ square units.

80. Find the vertical asymptote, if any, for the rational function $h(x) = \frac{3x^2 - 2x - 1}{4x - 1}$.

Solution:

- For $h(x) = \frac{3x^2 - 2x - 1}{4x - 1}$, a vertical asymptote occurs where the denominator is zero (assuming the numerator is not also zero at that point, in which case it might be a hole instead).
- Set the denominator equal to zero and solve for x : $4x - 1 = 0$
- $x = \frac{1}{4}$
- So, there is a vertical asymptote at $x = \frac{1}{4}$.

81. A garden planning app allocates a certain number of pixels on screen to represent lengths in a garden layout. If a user has a budget that allows for 600 pixels of length to design their rectangular garden, express the area A of the garden as a function of the width w, in pixels, of the rectangle.

Solution:

- If the length of the rectangular garden is represented by $600 - w$ pixels, where w is the width in pixels, then the area A of the rectangle as a function of width w is:
  $A(w) = w \cdot (600 - w)$
  $A(w) = 600w - w^2$
- This quadratic function represents the area in pixels squared.

82. If $x = 3$ is a real zero of the polynomial $g(x) = x^3 - 9x^2 + 27x - 27$, write $g(x)$ as a product of linear factors.

Solution:

- Given that $x=3$ is a zero, we know that $(x-3)$ is a factor.
- $g(x)=(x-3)(Ax^2+Bx+C)$
- Since the polynomial is cubic and we are given one real zero, there should be two more zeros that can be found by polynomial division or factoring. In this case, since the coefficients suggest a pattern, we can predict that the other zeros are also 3 (as it's the sum of the roots taken one at a time for a cubic equation with roots of the same value). Thus, we get:
- $g(x)=(x-3)(x-3)(x-3)=(x-3)^3$

83. Find the inverse of the matrix $B=\begin{bmatrix} 2 & -3 \\ 0 & 6 \end{bmatrix}$ if it exists.

Solution:

- The inverse of a 2x2 matrix $B=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is given by: $B^{-1}=\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
- Applying this to matrix B : $B^{-1}=\frac{1}{(2)(6)-(0)(-3)}\begin{bmatrix} 6 & 3 \\ 0 & 2 \end{bmatrix}$

  $B^{-1}=\frac{1}{12}\begin{bmatrix} 6 & 3 \\ 0 & 2 \end{bmatrix}=\begin{bmatrix} 0.5 & 0.25 \\ 0 & 0.1667 \end{bmatrix}$

84. A deposit of 15,000 is made in an account that earns 4.5% interest compounded monthly. The balance in the account after m months is given by the sequence $b_m=15{,}000\left(1+\frac{0.045}{12}\right)^m$. Find the balance in the account after 3 years.

Solution:

- Using the compound interest formula: $b_m=15{,}000\left(1+\frac{0.045}{12}\right)^{12\cdot 3}$

- To find the balance after 3 years (which is 36 months): $b_{36} = 15,000\left(1+\frac{0.045}{12}\right)^{36}$
- This calculation requires a calculator to evaluate.

85. If $\log_5(x+2)-\log_5(x-2)$**, then x lies in which of the following intervals?**

(A) $(2,\infty)$

(B) $(0,2)$

(C) $(-\infty,-2)$

(D) $(-2,0)$

Solution:

- The difference of two logs with the same base can be written as the log of a quotient: $\log_5 \frac{x+2}{x-2}$
- For the logarithm to be defined, the argument must be positive: $\frac{x+2}{x-2} > 0$
- This inequality will be true when both the numerator and denominator are positive (which happens when $x > 2$) or both are negative (which is never true since $x+2$ is always greater than $x-2$).
- Therefore, the interval is $x > 2$, which corresponds to choice (A).

86. If θ **is an angle in standard position and its terminal side passes through the point** $P(-0.5, 0.5)$ **on the unit circle, which of the following is a possible value radian value of** θ **to the nearest hundredth?**

(A) 2.36

(B) 3.14

(C) 4.71

(D) 5.50

Solution:

- Since $P(-0.5, 0.5)$ lies in the second quadrant and the coordinates are equal in magnitude, we can infer that θ is $\frac{3\pi}{4}$ radians (135 degrees).
- But since we want the value to the nearest hundredth, using a calculator we find that $\theta \approx 2.36$ radians. Thus, the answer is (A) 2.36.

87. A company finds that the revenue R, in dollars, from selling q units of a product is given by the revenue function $R(q) = 3q^2 - 18q + 500$. Determine the number of units sold that will maximize the revenue.

Solution:

- The revenue function is a quadratic function of the form R(q) = aq^2 + bq + c.
- To find the number of units that maximize revenue, we need to find the vertex of the parabola. The q-coordinate of the vertex can be found using the formula -b/(2a).
- In this case, a = 3 and b = -18. So, the q-coordinate is -(-18)/(2*3) = 18/6 = 3.
- Therefore, selling 3 units will maximize the revenue.

88. An object is thrown vertically upward, and its height t seconds after it is thrown is given by the equation $H(t) = 5 + 32t - 16t^2$. Calculate the average velocity of the object over the interval from t = 1 to t = 3 seconds.

Solution:

- To find the average velocity over a time interval, we calculate the change in height divided by the change in time.
- First, find the height at t = 1: $H(1) = 5 + 32(1) - 16(1)^2 = 5 + 32 - 16 = 21$ meters.
- Then, find the height at t = 3: $H(3) = 5 + 32(3) - 16(3)^2 = 5 + 96 - 144 = -43$ meters.
- The change in height is $H(3) - H(1) = -43 - 21 = -64$ meters.
- The change in time is 3 - 1 = 2 seconds.
- Therefore, the average velocity is -64 meters / 2 seconds = -32 meters per second.

89. Solve the inequality $\frac{3x+1}{2x+3} > 4$.

Solution:

- Begin by moving all terms to one side: $\frac{3x+1}{2x+3}-4>0$.
- Combine the terms over a common denominator: $\frac{3x+1-4(2x+3)}{2x+3}>0$.
- Simplify the numerator: $\frac{3x+1-8x-12}{2x+3}>0$ which simplifies to $\frac{-5x-11}{2x+3}>0$.
- Find the critical points by setting the numerator and denominator equal to zero: $-5x-11=0$ and $2x+3=0$.
- Solve for x: $x=-11/5$ and $x=-3/2$.
- Test intervals around the critical points to find where the inequality is true.
- After testing, you would find that the solution set is $x\in(-\infty,-3/2)\cup(-11/5,\infty)$.

90. Given the values of a function g(y) at various points, determine between which consecutive values of y does g(y) change sign, thus indicating the presence of a root.

y	-2	0	2	4
g(y)	7.14	-12.32	-0.48	13.92

Solution:

- Look for intervals where the function values change from positive to negative or vice versa.
- From y = -2 to y = 0, g(y) changes from positive to negative, indicating a root in the interval (-2, 0).
- From y = 2 to y = 4, g(y) changes from negative to positive, indicating a root in the interval (2, 4).
- Thus, there may be roots in the intervals (-2, 0) and (2, 4).

91. Determine the domain of the function $g(x)=\sqrt{m-x}-\frac{1}{\sqrt{n-x}}$, where $0<m<n$, $0<x<n$.

Solution:

- For $\sqrt{m-x}$ to be real, $m-x\geq 0$. Therefore, $x\leq m$.
- For $\frac{1}{\sqrt{n-x}}$ to be real and non-zero, $n-x>0$. Therefore, $x<n$.

- Since $0 < m < n$ and x must be less than n but not more than m , the domain of $g(x)$ is $(0, m]$.

92. Calculate the range of values for y for which the equation $3^{y^2} + 3^y - 4 = 0$ has real solutions, rounding your answer to 2 decimal places.

Solution:

- Rewrite the equation with a substitution $u = 3^y$ which gives us $u^2 + u - 4 = 0$.
- Solve for u using the quadratic formula, or factoring if possible.
- Once you have the values for u , take the logarithm base 3 to find y : $y = \log_3(u)$.
- Determine the values of y that are real and provide the range of those values.

93. If the pattern of growth for a certain species of bacteria doubles every hour, and there are 500 bacteria at time 0, estimate the population of the bacteria after 6 hours.

Solution:

- The growth of the bacteria follows an exponential pattern.
- If the population doubles every hour, then the population after t hours is given by $P(t) = 500 \times 2^t$.
- After 6 hours, the population will be $P(6) = 500 \times 2^6 = 500 \times 64 = 32000$ bacteria.

94. A box with a square base and open top is to be constructed from a square piece of cardboard with sides of 24 inches by removing equal squares of side y at each corner and folding up the flaps. What should be the side length of the squares cut from each corner to maximize the volume of the box?

Solution:

- The side length of the cardboard is 24 inches, so after cutting out squares of side y, the new dimensions of the base will be $(24 - 2y) \times (24 - 2y)$.
- The height of the box will be y inches.
- The volume V of the box can be expressed as $V(y) = y(24 - 2y)(24 - 2y)$.

- To find the maximum volume, we need to find the derivative V'(y) and set it to zero to find the critical points.
- Once you have the critical points, determine which value of y gives the maximum volume by either using the second derivative test or analyzing the sign of the first derivative around the critical points.
- The value of y that maximizes the volume will be the length of the side of the squares that should be cut from each corner.

95. Let $T_3 : \mathbb{R}^2 \to \mathbb{R}^2$ be a reflection about the y-axis, and let $T_4 : \mathbb{R}^2 \to \mathbb{R}^2$ be a dilation with factor k = -2. Find the standard matrix for the composition $T_4 \circ T_3$ on $\mathbb{R}^2$.

Solution:

- The reflection about the y-axis can be represented by the matrix $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$.
- The dilation with factor k = -2 can be represented by the matrix $\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$.
- The composition $T_4 \circ T_3$ means we apply T_3 first, followed by T_4.
- The standard matrix for the composition is found by multiplying the matrices for T_3 and T_4: $\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$.
- Perform the matrix multiplication to get the standard matrix for the composition $T_4 \circ T_3$.

96. A linear function Q is used to model the quantity, in thousands, of a certain product sold as a function of its price x , in dollars. It is known that $Q(10) = 40$ and $Q(15) = 25$. Based on this model, which of the following is true?

A) For each dollar increase in price, the quantity sold increases by approximately 3000 units.

B) For each dollar increase in price, the quantity sold decreases by approximately 1500 units.

C) For each dollar increase in price, the quantity sold decreases by approximately 3000 units.

D) For each dollar increase in price, the quantity sold increases by approximately 1500 units.

Solution:

- Find the change in quantity corresponding to the change in price. This can be done by subtracting the quantities and prices at the given points:
  - $\Delta Q = Q(15) - Q(10) = 25 - 40 = -15$ (in thousands)
  - $\Delta x = 15 - 10 = 5$ (in dollars)
- Calculate the rate of change (slope) of the quantity with respect to the price:
  - Slope $m = \frac{\Delta Q}{\Delta x} = \frac{-15}{5} = -3$ (in thousands per dollar)
- Since the slope is negative, the quantity sold decreases as the price increases. And because the slope is -3, the quantity sold decreases by 3000 units for each dollar increase in price.
- The correct statement based on this model is: C) For each dollar increase in price, the quantity sold decreases by approximately 3000 units.

97. The function g is given by $g(z) = z^4 - z^2$. Which of the following statements is true and supports the claim that g is an even function and not an odd function?

A) $g(0) = g(-0)$

B) $g(2) = g(-2)$

C) $-g(2) = g(-2)$

D) $g(2) = -g(-2)$

Solution:

- For a function to be even, the equation g(z) = g(-z) must hold true for every z in the domain of g .
- Apply the definition of an even function to the given function:
  - $g(z) = z^4 - z^2$
  - $g(-z) = (-z)^4 - (-z)^2 = z^4 - z^2$
- Since $g(z)$ is equal to $g(-z)$, it confirms that g is an even function.
- The correct statement that supports the claim that g is an even function is: B) $g(2) = g(-2)$

98. The polynomial function m is given by $m(x) = x^4 - 6x^3 + 8$. Which of the following describes the behavior of m as the input values increase without bound?

A) The output values decrease without bound.

B) The output values increase without bound.

C) The output values sometimes increase without bound and sometimes decrease without bound.

D) The output values get closer to 8.

Solution:

- Identify the leading term of the polynomial m(x) , which is x^4 in this case.
- The behavior of the polynomial for large positive values of x (*as* x increases without bound) is determined by the leading term, because as x becomes very large, the leading term will dominate the behavior of the polynomial.
- For positive values of x getting larger and larger, x^4 will increase without bound because it is a positive term with an even power.
- The correct statement that describes the behavior of m as the input values increase without bound is: B) The output values increase without bound.

99. The table gives values for the function g at selected values of x . Which of the following conclusions with reason is consistent with the values in the table?

x	1	2	3	4	6
g(x)	3	8	15	24	35

A) The graph of g is concave up because the second differences of $g(x)$ are constant and positive.

B) The graph of g is concave down because the second differences of $g(x)$ are constant and positive.

C) The graph of g is linear because the first differences of $g(x)$ are constant.

D) The graph of g is concave up because the first differences of $g(x)$ are increasing.

Solution:

- Calculate the first differences of $g(x)$ by finding the differences between consecutive $g(x)$ values.
    - $g(2)-g(1)=8-3=5$
    - $g(3)-g(2)=15-8=7$
    - $g(4)-g(3)=24-15=9$
    - $g(5)-g(4)=35-24=11$
- Observe if the first differences are constant. Since they are not constant (5, 7, 9, 11), g is not linear.
- Calculate the second differences of $g(x)$ to determine the concavity.
    - $7-5=2$
    - $9-7=2$
    - $11-9=2$
- Since the second differences are constant and positive, the graph of g is concave up.
- The correct statement consistent with the values in the table is:
    - The graph of g is concave up because the second differences of $g(x)$ are constant and positive.

100. The polynomial function h is given by $h(x)=cx^d$, where c is a non-zero constant and d is a positive integer. It is known that $\lim_{x\to\infty} h(x)=\infty$ and $\lim_{x\to-\infty} h(x)=-\infty$. Which of the following statements must be true?

A) The value of c must be positive, because as x increases without bound or decreases without bound, the end behaviors of h are different.

B) The value of c must be negative, because as x increases without bound or decreases without bound, the end behaviors of h are the same.

C) The value of d must be even, because as x increases without bound or decreases without bound, the end behaviors of h are the same.

D) The value of d must be odd, because as x increases without bound or decreases without bound, the end behaviors of h are different.

Solution:

- Consider the given limits: $\lim_{x\to\infty} h(x)=\infty$ and $\lim_{x\to-\infty} h(x)=-\infty$. This tells us about the end behavior of the polynomial as x goes to positive and negative infinity.

- Recognize that for $h(x) = cx^d$, if c is positive and d is odd, $h(x)$ will approach infinity as x approaches infinity and negative infinity as x approaches negative infinity.
- The behavior described in the limits matches the behavior of a polynomial with an odd degree and a positive leading coefficient.
- The correct statement that must be true is:D) The value of d must be odd, because as x increases without bound or decreases without bound, the end behaviors of h are different.

101. The function L models the level of a certain medication in the bloodstream and is given by $L(t) = \frac{300t}{1+3t}$ for $t \geq 0$, where t is measured in hours since the medication was taken. Which of the following describes the level of the medication in the bloodstream as time increases?

A) The level decreases and approaches a value of 0 mg/L.

B) The level increases and approaches a value of 100 mg/L.

C) The level increases and approaches a value of 300 mg/L.

D) The medication level increases without bound.

Solution:

- To determine the long-term behavior of L(t) , examine the horizontal asymptote as t approaches infinity.
- As t increases without bound, the 3t in the denominator grows much larger than the 1, and thus the $1+3t$ term behaves like 3t .
- Therefore, the function $L(t)$ behaves like $\frac{300t}{3t}$ as t becomes very large.
- Simplify this to $\frac{300}{3}$, which equals 100.
- This indicates that the level of the medication approaches 100 mg/L as time increases.
- The correct description of the level of the medication as time increases is: B) The level increases and approaches a value of 100 mg/L.

102. The function s is given by $s(x) = \frac{x+2}{x-4}$. What are all solutions to $s(x) < 0$?

A) $x < -2$ and $x > 4$

B) $x < 4$ only

C) $x > -2$ only

D) $x > -2$ and $x < 4$

Solution:

- For the rational function $s(x)$ to be less than 0, the numerator and denominator must have opposite signs.
- The numerator $x + 2$ is negative when $x < -2$.
- The denominator $x - 4$ is negative when $x < 4$.
- For $s(x)$ to be negative, the numerator must be positive while the denominator is negative, or vice versa.
- Therefore, $s(x) < 0$ when $x > -2$ and $x < 4$.
- The correct interval that describes all solutions for which $s(x) < 0$ is:
- D) $x > -2$ and $x < 4$.

103. The zeros of a rational function h are 2 and -3. Which of the following expressions could define h(x) ?

A) $\frac{(x-1)(x+4)}{(x-2)(x+3)}$

B) $\frac{(x-2)(x+3)}{(x-1)(x+4)}$

C) $\frac{(x-2)(x+3)}{(x+2)(x-3)}$

D) $\frac{(x+2)(x-3)}{(x-2)(x+3)}$

Solution:

- For a rational function h(x) = \frac{N(x)}{D(x)} , the zeros are the values of x for which $N(x) = 0$ and $D(x) \neq 0$.
- To have zeros at $x = 2$ and $x = -3$, the numerator must be zero when $x = 2$ and $x = -3$, and these cannot be zeros of the denominator.
- Therefore, $(x-2)$ and $(x+3)$ must be factors of the numerator.

- To ensure these are not canceled out by the denominator, $(x-2)$ and $(x+3)$ must not be factors of the denominator.
- The correct expression that would define $h(x)$ with zeros at 2 and -3 is: B) $\frac{(x-2)(x+3)}{(x-1)(x+4)}$

104. For the function h , it is known that $h(2)=0$ and $h(5)=-3$. The function k is given by $k(x)=h(x-3)$. Which of the following must be a solution to $k(x)=0$?

A) $x=-1$

B) $x=1$

C) $x=5$

D) $x=8$

Solution:

- The function $k(x)$ is a horizontal shift of the function $h(x)$ to the right by 3 units.
- If $h(2)=0$, then $k(x)=0$ when $x-3=2$.
- Solving for x gives $x=2+3=5$.
- So, the value of x that must be a solution to $k(x)=0$ is:C) $x=5$

105. Which of the following functions has the same end behavior as the rational function q given by $q(x)=\frac{3x^2+4x-7}{5x^2-x+2}$?

A) $f(x)=\frac{3}{5}$

B) $g(x)=1$

C) $h(x)=-\frac{3}{5}$

D) $k(x)=x$

Solution:

- To determine the end behavior of the rational function $q(x)$, look at the leading terms in the numerator and denominator.

- As x approaches infinity or negative infinity, the behavior of $q(x)$ is dominated by the terms $3x^2$ in the numerator and $5x^2$ in the denominator.
- The coefficients of x^2 in both the numerator and denominator indicate that the function approaches the ratio of these coefficients.
- Simplify this ratio to get $\frac{3}{5}$.
- The function that has the same end behavior as the given rational function is:A) $f(x)=\frac{3}{5}$

106. The rational function m is given by $m(x)=\frac{(x-1)^2(x+3)(x-4)}{x^2(x-1)(x+2)^3}$. For which of the following values of x does the graph of m have vertical asymptotes?

A) $x=1$ and $x=-2$ only

B) $x=0$ and $x=-2$ only

C) $x=0$, $x=1$, and $x=-2$

D) $x=0$, $x=1$, and $x=4$

Solution:

- Vertical asymptotes of a rational function occur where the denominator is zero but the numerator is not zero.
- Identify the values of x that cause the denominator $x^2(x-1)(x+2)^3$ to be zero, which are $x=0$, $x=1$, and $x=-2$.
- However, if these values also make the numerator $(x-1)^2(x+3)(x-4)$ zero, then they are not vertical asymptotes, but rather holes in the graph.
- Since $x=1$ is a factor of both the numerator and denominator, it is not a vertical asymptote. It will be a hole in the graph because the factor $(x-1)$ cancels.
- Thus, we only consider $x=0$ and $x=-2$ for the vertical asymptotes, provided they don't cancel out.
- The correct set of x values where $m(x)$ has vertical asymptotes is: B) $x=0$ and $x=-2$ only.

107. The domain of the function f(x) is $-4 \le x \le 16$. If the function k(x) is given by $k(x) = f\left(\frac{x}{3}\right) + 1$, what is the domain of k ?

Solution:

- To find the domain of k, we'll consider the domain of f and apply the transformation given by $k(x) = f\left(\frac{x}{3}\right) + 1$.
- The domain of f is given by $-4 \le x \le 16$. For the function $k(x)$, we are concerned with the argument of f, which is $\frac{x}{3}$. We need to find the range of x values such that $\frac{x}{3}$ falls within the domain of f.
- Let's solve for x: $-4 \le \frac{x}{3} \le 16$
- Now, we multiply all parts by 3: $-4 \times 3 \le x \le 16 \times 3 \Rightarrow -12 \le x \le 48$
- So the domain of k is: $-12 \le x \le 48$

108. The domain of the function f(x) is . $-4 \le x \le 16$.. If the function k(x) is given by $k(x) = f\left(\frac{x}{3} + 1\right)$, what is the domain of k ?

Solution:

- To find the domain of k, we'll consider the domain of f and apply the transformation given by $k(x) = f\left(\frac{x}{3} + 1\right)$.
- Let's solve for x: $-4 \le \frac{x}{3} + 1 \le 16$
- Now, solve for x: $-15 \le x \le 45$
- So the domain of k is: $-15 \le x \le 45$